

Chapter 7: A Tale of Three Intersecting Lines

In-text questions (Page 146)

Q. What happens when the three vertices lie on a straight line?

Solution:

When the three vertices lie on a straight line, they are said to be collinear. In this case, they do not form a closed shape, and hence, a triangle cannot be formed.

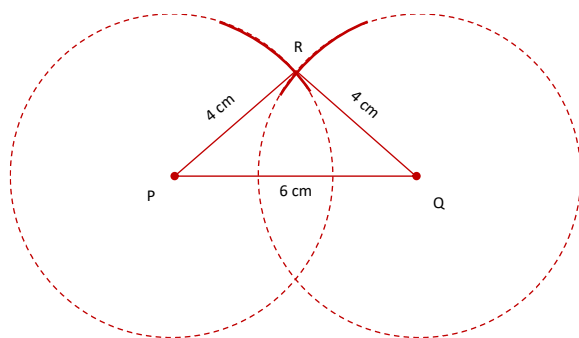
Construct (Page 150)

Q. Construct triangles having the following sidelengths (all the units are in cm):

- (a) 4, 4, 6
- (b) 3, 4, 5
- (c) 1, 5, 5
- (d) 4, 6, 8
- (e) 3.5, 3.5, 3.5

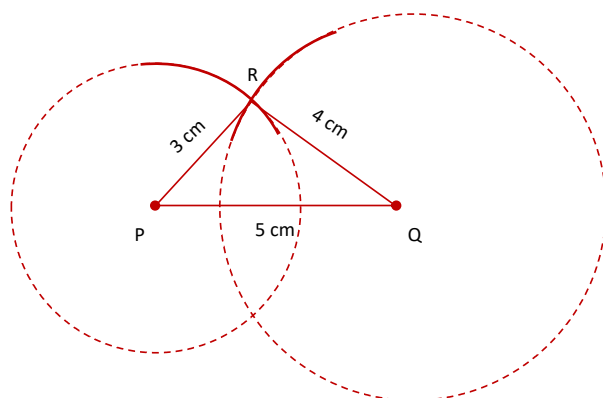
Solution:

- (a) 4, 4, 6



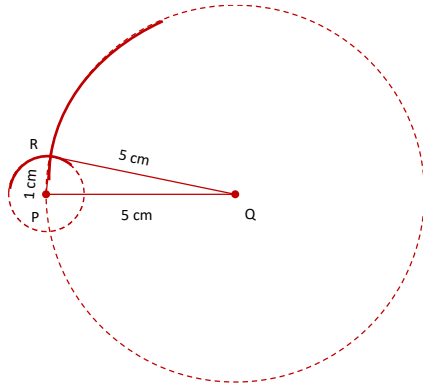
- (i) Draw the base PQ using one of the given side lengths. Let $PQ = 6$ cm.
- (ii) From points P and Q, draw arcs of radius 4 cm each. Let the arcs intersect at point R.
- (iii) The point of intersection R is the required third vertex. Join PR and QR to form $\triangle PQR$.

- (b) 3, 4, 5



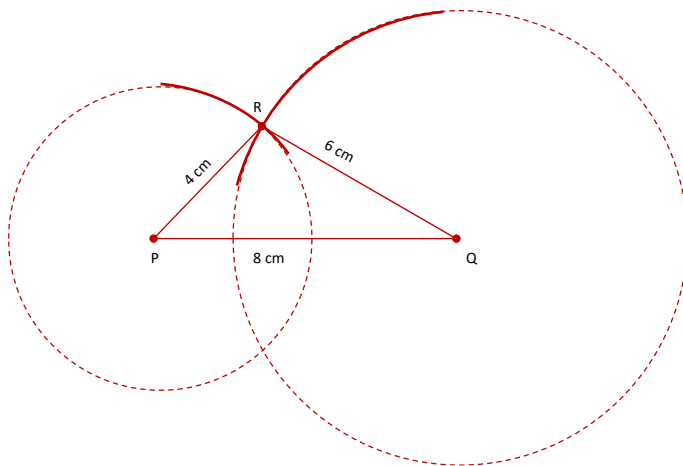
- (i) Draw the base PQ with one of the side lengths. Let $PQ = 5$ cm.
- (ii) From P, draw a long arc of radius 3 cm.
- (iii) From Q, draw an arc of radius 4 cm intersecting the first arc at point R.
- (iv) The point R is the required third vertex. Join PR and QR to get ΔPQR .

(c) 1, 5, 5



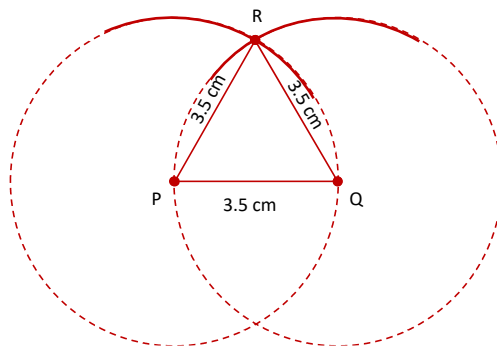
- (i) Construct the base PQ with one of the side lengths. Let $PQ = 5$ cm.
- (ii) From P, draw long arc of radius 1 cm.
- (iii) From Q, draw arc of radius 5 cm intersecting the first arc at point R.
- (iv) The point R is the required third vertex. Join PR and QR to get ΔPQR .

(d) 4, 6, 8



- (i) Construct the base PQ with one of the side lengths. Let $PQ = 8$ cm.
- (ii) From P, draw a long arc of radius 4 cm.
- (iii) From Q, draw an arc of radius 6 cm intersecting the first arc at point R.
- (iv) The point R is the required third vertex. Join PR and QR to get ΔPQR .

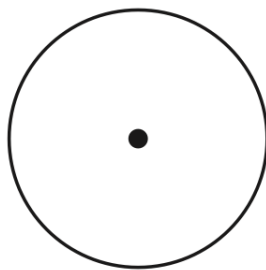
(e) 3.5, 3.5, 3.5



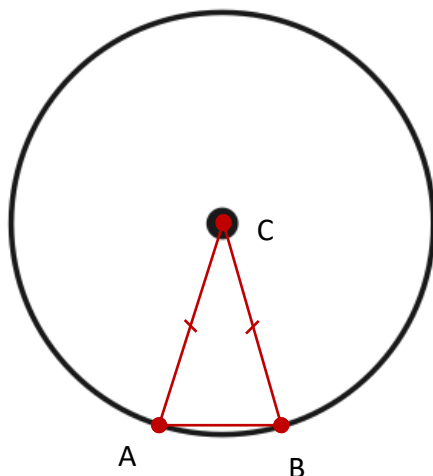
- (i) Construct the base PQ of length 3.5 cm.
- (ii) From points P and Q, draw arcs of radius 3.5 cm each. Let the arcs intersect at point R.
- (iii) The point R is the required third vertex. Join PR and QR to get $\triangle PQR$.

Figure it Out (Page 150)

1. Use the points on the circle and/or the centre to form isosceles triangles.

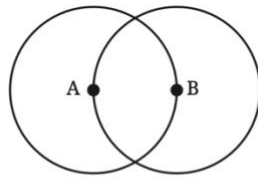


Solution:

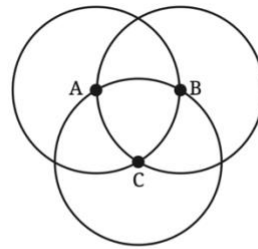


- (i) Take any two points A and B on the circle and connect them by drawing a chord.
- (ii) Draw lines from the center of the circle C to each of these points.
- (iii) The triangle formed by these two radii and the chord is an isosceles triangle with two sides equal to the radius of the circle.

2. Use the points on the circles and/or their centres to form isosceles and equilateral triangles. The circles are of the same size.

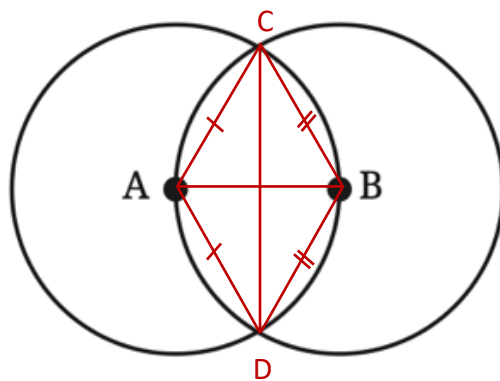


A and B are the centres of circles of the same size



A, B, and C are the centres of circles of the same size

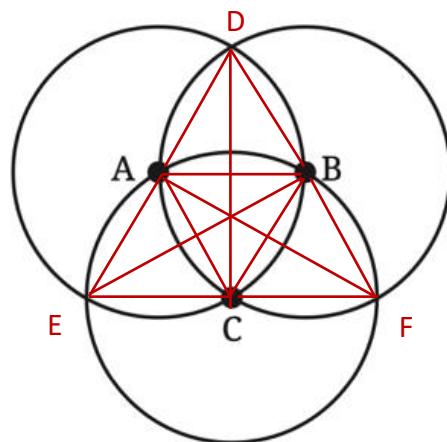
Solution:



(i) Join the points of intersection and the centres of the two circles to form isosceles and equilateral triangles.

Isosceles triangles: $\triangle ACD$ and $\triangle BCD$.

Equilateral triangles: $\triangle ACB$ and $\triangle ADB$.



(ii) Join the points of intersection and centres of the three circles to form isosceles and equilateral triangles.

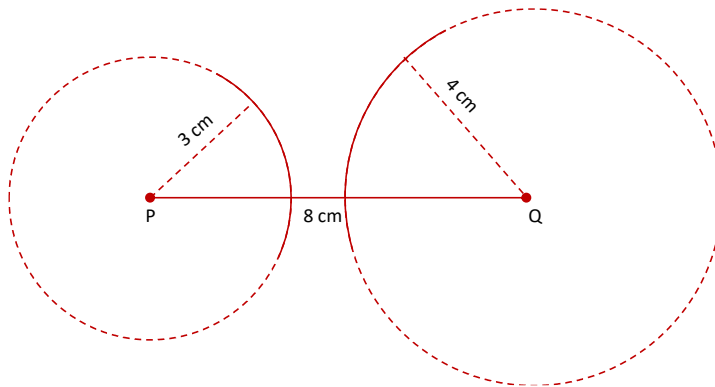
Isosceles triangles: $\triangle ADC$, $\triangle BDC$, $\triangle AEB$, $\triangle ECB$, $\triangle BAF$ and $\triangle CAF$.

Equilateral triangles: $\triangle DAB$, $\triangle ACB$, $\triangle AEC$ and $\triangle BCF$.

In-text questions (Page 148)

Q. Construct a triangle with sidelengths 3 cm, 4 cm, and 8 cm.
What is happening? Are you able to construct the triangle?

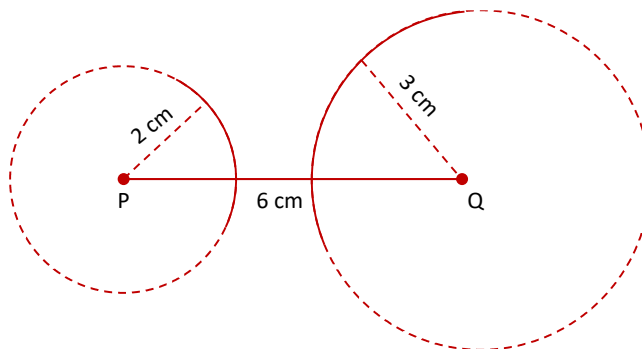
Solution:



From the base PQ of length 8 cm, arcs are drawn from points P and Q with radii 3 cm and 4 cm, respectively. However, these arcs do not intersect at any point. Therefore, it is not possible to construct a triangle with side lengths 3 cm, 4 cm, and 8 cm.

Q. Here is another set of lengths: 2 cm, 3 cm, and 6 cm. Check if a triangle is possible for these sidelengths.

Solution:

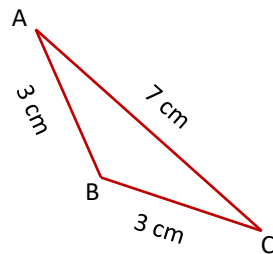


No, it is impossible to construct a triangle with side lengths 2 cm, 3 cm and 6 cm.

In-text questions (Page 153)

Q. Can we say anything about the existence of a triangle having sidelengths 3 cm, 3 cm and 7 cm? Verify your answer by construction.

Solution:



Rough Diagram

Consider the paths between A and B:

Direct path length = $AB = 3$ cm

Round about path length via vertex C = $AC + BC = 7 + 3 = 10$ cm

Consider the paths between B and C:

Direct path length = $BC = 3$ cm

Round about path length via vertex A = $AB + AC = 3 + 7 = 10$ cm

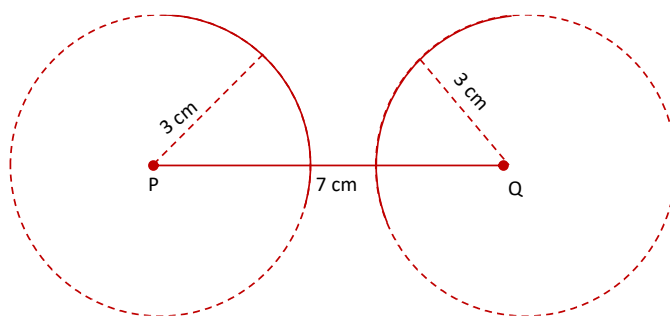
Consider the paths between A and C:

Direct path length = $AC = 7$ cm

Round about path length via vertex B = $AB + CB = 3 + 3 = 6$ cm

Since the direct path is longer than the roundabout path here. Therefore, such a triangle doesn't exist

Additionally, a triangle with the given sidelengths is not possible as the two arcs from the end points of the base do not meet to provide a third vertex.



Q. “In the rough diagram in Fig. 7.4, is it possible to assign lengths in a different order such that the direct paths are always coming out to be shorter than the roundabout paths? If this is possible, then a triangle might exist.”

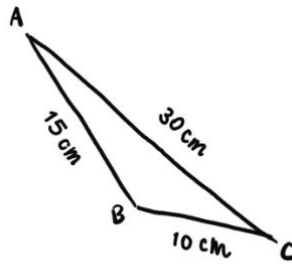


Fig. 7.4

Solution:

No, rearranging the sides 10 cm, 15 cm, and 30 cm won't help to form a triangle because the largest side (30 cm) is greater than the sum of the other two sides ($10\text{ cm} + 15\text{ cm} = 25\text{ cm}$), which makes triangle formation impossible.

Q. Is such rearrangement of lengths possible in the triangle?

Solution:

No, the rearrangement of the lengths 10 cm, 15 cm, and 30 cm can't help in the formation of a triangle because 30 cm is always going to be greater than $10 + 15 = 25\text{ cm}$, regardless of the order of the sides.

Figure it Out (Page 154)

1. We checked by construction that there are no triangles having sidelengths 3 cm, 4 cm and 8 cm; and 2 cm, 3 cm and 6 cm. Check if you could have found this without trying to construct the triangle.

Solution:

(a) Let $AB = 3\text{ cm}$, $BC = 4\text{ cm}$ and $AC = 8\text{ cm}$.

Consider the paths between A and B:

Direct path length = $AB = 3\text{ cm}$

Round about path length via vertex C = $AC + BC = 8 + 4 = 12\text{ cm}$.

Thus, the direct path length is shorter than the roundabout path length.

Consider the paths between B and C:

Direct path length = $BC = 4\text{ cm}$

Round about path length via vertex A = $AB + AC = 3 + 8 = 11\text{ cm}$.

Thus, the direct path length is shorter than the roundabout path length.

Consider the paths between A and C:

Direct path length = $AC = 8\text{ cm}$

Round about path length via vertex B = $AB + BC = 3 + 4 = 7\text{ cm}$.

Here, the direct path length is longer than the roundabout path length.

So, the triangle with the given side lengths doesn't exist.

(b) Let $AB = 2$ cm, $BC = 3$ cm and $AC = 6$ cm.

Consider the paths between A and B:

Direct path length = $AB = 2$ cm

Round about path length via vertex C = $AC + BC = 6 + 3 = 9$ cm.

Thus, the direct path length is shorter than the roundabout path length.

Consider the paths between B and C:

Direct path length = $BC = 3$ cm

Round about path length via vertex A = $AB + AC = 2 + 6 = 8$ cm.

Thus, the direct path length is shorter than the roundabout path length.

Consider the paths between A and C:

Direct path length = $AC = 6$ cm

Round about path length via vertex B = $AB + BC = 2 + 3 = 5$ cm.

Here, the direct path length is longer than the roundabout path length.

So, the triangle with the given side lengths doesn't exist.

2. Can we say anything about the existence of a triangle for each of the following sets of lengths?

(a) 10 km, 10 km and 25 km

(b) 5 mm, 10 mm and 20 mm

(c) 12 cm, 20 cm and 40 cm

You would have realised that using a rough figure and comparing the direct path lengths with their corresponding roundabout path lengths is the same as comparing each length with the sum of the other two lengths. There are three such comparisons to be made.

Solution:

(a) If the direct path is 25 km long, then the roundabout path is $10 \text{ km} + 10 \text{ km} = 20 \text{ km}$.

Since the direct path cannot be longer than the roundabout path.

Therefore, 10 km, 10 km, and 25 km can't be the side lengths of a triangle.

(b) If the direct path is 20 mm long, then the roundabout path is $5 \text{ mm} + 10 \text{ mm} = 15 \text{ mm}$.

Since the direct path cannot be longer than the roundabout path.

Therefore, 5 mm, 10 mm, and 20 mm can't be the side lengths of a triangle.

(c) If the direct path is 40 cm long, then the roundabout path is $12 \text{ cm} + 20 \text{ cm} = 32 \text{ cm}$.

Since the direct path cannot be longer than the roundabout path.

Therefore, 12 cm, 20 cm, and 40 cm can't be the side lengths of a triangle.

3. For each set of lengths seen so far, you might have noticed that in at least two of the comparisons, the direct length was less than the sum of the other two (if not, check again!).

For example, for the set of lengths 10 cm, 15 cm and 30 cm, there are two comparisons where this happens:

$$10 < 15 + 30$$

$$15 < 10 + 30$$

But this doesn't happen for the third length: $30 > 10 + 15$.

Will this always happen? That is, for any set of lengths, will there be at least two comparisons where the direct length is less than the sum of the other two? Explore for different sets of lengths.

Solution:

Yes, for any set of lengths, there will always be at least two comparisons where the direct length is less than the sum of the other two. If two out of three comparisons have a direct length smaller than the sum of the other two, then such a triangle doesn't exist.

However, if all three comparisons have direct length smaller than the sum of the other two, then such a triangle exists.

Let us consider some examples:

(a) 5 cm, 7 cm and 9 cm

$$5 + 7 > 9$$

$$7 + 9 > 5$$

$$9 + 5 > 7$$

Here, all three comparisons have a direct length smaller than the sum of the other two lengths. Hence, a triangle with the given sidelengths exists.

(b) 2 cm, 3 cm and 6 cm

$$2 + 3 < 6$$

$$3 + 6 > 2$$

$$6 + 2 > 3$$

Here, only two comparisons have a direct length smaller than the sum of the other two lengths. Hence, a triangle with the given sidelengths doesn't exist.

(c) 7 cm, 15 cm and 30 cm

$$7 + 15 < 30$$

$$15 + 30 > 7$$

$$30 + 7 > 15$$

Here, all three comparisons have a direct length smaller than the sum of the other two lengths. Hence, a triangle with the given sidelengths doesn't exist.

Q. Further, for a given set of lengths, is it possible to identify which lengths will immediately be less than the sum of the other two, without calculations?

Solution:

Yes, it is possible to identify which length will be immediately less than the sum of the other two by simply arranging the direct lengths in increasing order.

Figure it out (Page 156)

1. Which of the following lengths can be the sidelengths of a triangle? Explain your answers. Note that for each set, the three lengths have the same unit of measure.

- (a) 2, 2, 5 (b) 3, 4, 6
(c) 2, 4, 8 (d) 5, 5, 8
(e) 10, 20, 25 (f) 10, 20, 35
(g) 24, 26, 28

We observe from the previous problems that whenever there is a set of lengths satisfying the triangle inequality (each length $<$ sum of the other two lengths), there is a triangle with those three lengths as sidelengths.

Solution:

(a) 2, 2, 5

$$2 + 2 < 5$$

$$2 + 5 > 2$$

$$5 + 2 > 2$$

Since, the lengths don't follow the triangle inequality. Therefore, they can't be the side lengths of a triangle.

b) 3, 4, 6

$$3 + 4 > 6$$

$$4 + 6 > 3$$

$$6 + 3 > 4$$

Since, the lengths follow the triangle inequality. Therefore, they can be the side lengths of a triangle.

c) 2, 4, 8

$$2 + 4 < 8$$

$$4 + 8 > 2$$

$$8 + 2 > 4$$

Since, the lengths don't follow the triangle inequality. Therefore, they can't be the side lengths of a triangle.

d) 5, 5, 8

$$5 + 5 > 8$$

$$5 + 8 > 5$$

$$8 + 5 > 5$$

Since, the lengths follow the triangle inequality. Therefore, they can be the side lengths of a triangle.

e) 10, 20, 25

$$10 + 20 > 25$$

$$20 + 25 > 10$$

$$25 + 10 > 20$$

Since, the lengths follow the triangle inequality. Therefore, they can be the side lengths of a triangle.

(f) 10, 20, 35

$$10 + 20 < 35$$

$$20 + 35 > 10$$

$$35 + 10 > 20$$

Since, the lengths don't follow the triangle inequality. Therefore, they can't be the side lengths of a triangle.

g) 24, 26, 28

$$24 + 26 > 28$$

$$26 + 28 > 24$$

$$28 + 24 > 26$$

Since, the lengths follow the triangle inequality. Therefore, they can be the side lengths of a triangle.

In-text questions (Page 159)

Q. How will the two circles turn out for a set of lengths that do not satisfy the triangle inequality? Find 3 examples of sets of lengths for which the circles:

(a) touch each other at a point,

(b) do not intersect.

Solution:

For a set of lengths that do not satisfy the triangle inequality, the two circles either touch each other at a point or do not intersect internally.

a) Circles touch each other at a point:

sum of the two smaller lengths = longest length

4 cm, 4 cm, 8 cm

3 cm, 3 cm, 6 cm

5 cm, 5 cm, 10 cm

b) Circles do not intersect:

sum of the two smaller lengths < longest length

5 cm, 10 cm, 18 cm

3 cm, 7 cm, 12 cm

4 cm, 8 cm, 15 cm

Q. Frame a complete procedure that can be used to check the existence of a triangle.

Solution:

(i) Arrange the given lengths in increasing order.

(ii) Check if each length is smaller than the sum of the other two lengths.

(iii) If yes, a triangle can be formed. If not, a triangle cannot be formed.

Figure it out (Page 159)

1. Check if a triangle exists for each of the following set of lengths:

- (a) 1, 100, 100 (b) 3, 6, 9
(c) 1, 1, 5 (d) 5, 10, 12

Solution:

- (a) 1, 100, 100
 $1 + 100 > 100$
 $100 + 100 > 1$
 $100 + 1 > 100$

Since, the lengths follow the triangle inequality. Therefore, they can be the side lengths of a triangle.

- (b) 3, 6, 9
 $3 + 6 = 9$
 $6 + 9 > 3$
 $9 + 3 > 6$

Since, the lengths don't follow the triangle inequality. Therefore, they can't be the side lengths of a triangle.

- (c) 1, 1, 5
 $1 + 1 < 5$
 $1 + 5 > 1$
 $5 + 1 > 1$

Since, the lengths don't follow the triangle inequality. Therefore, they can't be the side lengths of a triangle.

- (d) 5, 10, 12
 $5 + 10 > 12$
 $10 + 12 > 5$
 $12 + 5 > 10$

Since, the lengths follow the triangle inequality. Therefore, they can be the side lengths of a triangle.

2. Does there exist an equilateral triangle with sides 50, 50, 50? In general, does there exist an equilateral triangle of any sidelength? Justify your answer.

Solution:

Yes, an equilateral triangle with sides 50, 50, and 50 can exist because each side (50) is less than the sum of the other two sides ($50 + 50 = 100$), which satisfies the triangle inequality. Yes, an equilateral triangle can be constructed of any sidelength, satisfying the triangle inequality.

3. For each of the following, give at least 5 possible values for the third length so there exists a triangle having these as sidelengths (decimal values could also be chosen):

(a) 1, 100

(b) 5, 5

(c) 3, 7

Solution:

(a) For a triangle to exist, the sum of the two smaller lengths $>$ longest length.

For a triangle with sides 1 and 100, five valid possible values for the third side are: 99.1, 99.7, 100.3, 100.6, 100.8.

Because:

$1 + 99.1 > 100$; $1 + 99.7 > 100$; $1 + 100 > 100.3$; $1 + 100 > 100.6$ and $1 + 100 > 100.8$.

(b) For a triangle to exist, the sum of the two smaller lengths $>$ longest length.

For a triangle with sides 5 and 5, five valid possible values for the third side are: 1, 3, 4.5, 7, 9.9.

Because:

$1 + 5 > 5$; $3 + 5 > 5$; $5 + 4.5 > 5$; $5 + 5 > 7$ and $5 + 5 > 9.9$.

(c) For a triangle to exist, the sum of the two smaller lengths $>$ longest length.

For a triangle with sides 3 and 7, five valid possible values for the third side are: 4.1, 5, 6.5, 8, 9.9.

Because:

$3 + 4.1 > 7$; $3 + 5 > 7$; $3 + 6.5 > 7$; $3 + 7 > 8$ and $3 + 7 > 9.9$.

Figure it out (Page 161)

1. Construct triangles for the following measurements where the angle is included between the sides:

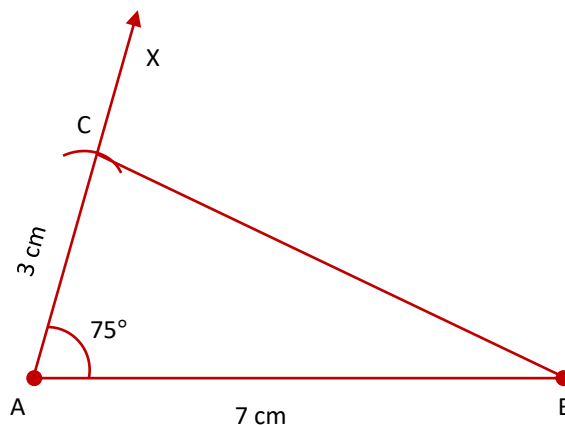
(a) 3 cm, 75° , 7 cm

(b) 6 cm, 25° , 3 cm

(c) 3 cm, 120° , 8 cm

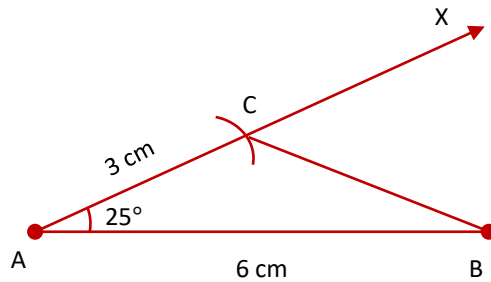
Solution:

(a) 3 cm, 75° , 7 cm



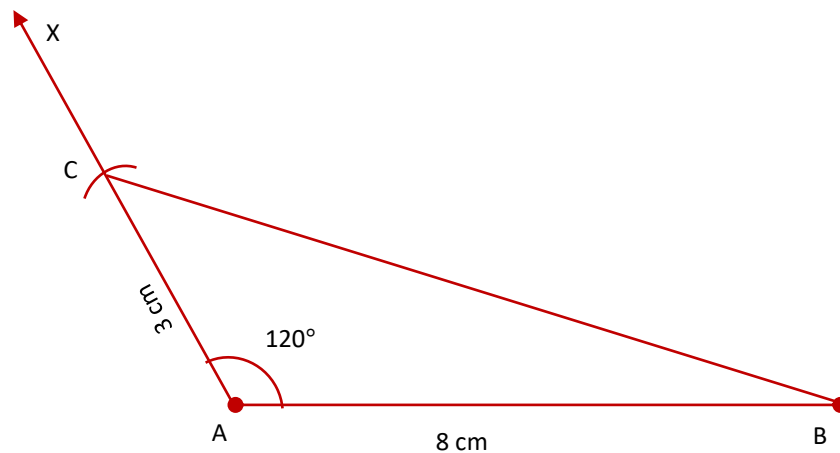
- (i) Construct side AB of length 7 cm.
- (ii) At point A, draw a ray AX making an angle of 75° with side AB.
- (iii) With A as the centre and radius 3 cm, draw an arc intersecting ray AX at point C.
- (iv) Join points B and C to form triangle $\triangle ABC$.

(b) 6 cm, 25° , 3 cm



- (i) Construct side AB of length 6 cm.
- (ii) At point A, draw a ray AX making an angle of 25° with side AB.
- (iii) With A as the centre and radius 3 cm, draw an arc intersecting ray AX at point C.
- (iv) Join points B and C to form triangle $\triangle ABC$.

(c) 3 cm, 120° , 8 cm



- (i) Construct side AB of length 8 cm.
- (ii) At point A, draw a ray AX making an angle of 120° with side AB.
- (iii) With A as the centre and radius 3 cm, draw an arc intersecting ray AX at point C.
- (iv) Join points B and C to form triangle $\triangle ABC$.

Figure it out (Page 162)

1. Construct triangles for the following measurements:

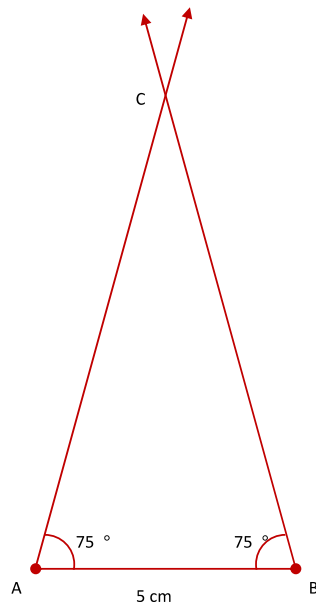
(a) 75° , 5 cm, 75°

(b) 25° , 3 cm, 60°

(c) 120° , 6 cm, 30°

Solution:

(a) 75° , 5 cm, 75°

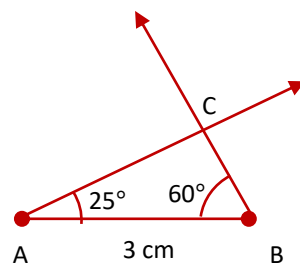


(i) Draw the base AB of length 5 cm.

(ii) Draw $\angle A$ and $\angle B$ both of measures 45° each.

(iii) The point of intersection of the two new line segments is the third vertex C.

(b) 25° , 3 cm, 60°

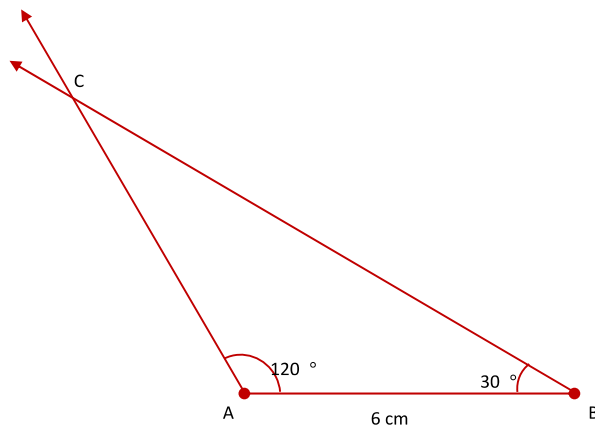


(i) Draw the base AB of length 3 cm.

(ii) Draw $\angle A$ and $\angle B$ of measures 25° and 60° respectively.

(iii) The point of intersection of the two new line segments is the third vertex C.

(c) 120° , 6 cm, 30°



- (i) Draw the base AB of length 6 cm.
- (ii) Draw $\angle A$ and $\angle B$ of measures 120° and 30° respectively.
- (iii) The point of intersection of the two new line segments is the third vertex C.

Figure it out (Page 163)

1. For each of the following angles, find another angle for which a triangle is (a) possible, (b) not possible. Find at least two different angles for each category:

- (a) 30°
- (b) 70°
- (c) 54°
- (d) 144°

Solution:

- (i) 30°

A triangle is possible when another angle is less than 150° . Examples of angles are 120° and 85° .

A triangle is not possible when another angle is greater than or equal to 150° . Examples of angles are 165° and 170° .

- (ii) 70°

A triangle is possible when another angle is less than 110° . Examples of angles are 100° and 67° .

A triangle is not possible when another angle is greater than or equal to 110° . Examples of angles are 135° and 150° .

- (iii) 54°

A triangle is possible when another angle is less than 126° . Examples of angles are 105° and 95° .

A triangle is not possible when another angle is greater than or equal to 126° . Examples of angles are 139° and 145° .

(iv) 144°

A triangle is possible when another angle is less than 36° . Examples of angles are 20° and 35° .

A triangle is not possible when another angle is greater than or equal to 36° . Examples of angles are 65° and 45° .

2. Determine which of the following pairs can be the angles of a triangle and which cannot:

(a) $35^\circ, 150^\circ$

(b) $70^\circ, 30^\circ$

(c) $90^\circ, 85^\circ$

(d) $50^\circ, 150^\circ$

Solution:

(a) $35^\circ + 150^\circ = 185^\circ$

$185^\circ > 180^\circ$.

Since the sum of these two angles is greater than 180° , they can't be the angles of a triangle.

(b) $70^\circ + 30^\circ = 100^\circ$

$100^\circ < 180^\circ$.

Since the sum of these two angles is less than 180° , they can be the angles of a triangle.

(c) $90^\circ + 85^\circ = 175^\circ$

$175^\circ < 180^\circ$.

Since the sum of these two angles is less than 180° , they can be the angles of a triangle.

(d) $50^\circ + 150^\circ = 200^\circ$

$200^\circ > 180^\circ$.

Since the sum of these two angles is greater than 180° , they can't be the angles of a triangle.

In-text questions (Page 164)

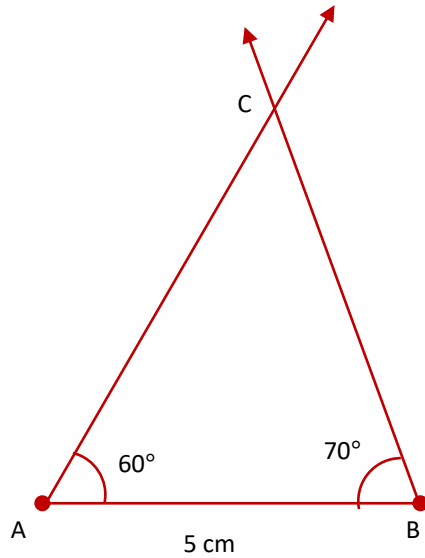
Q. Let us take two angles, say 60° and 70° , whose sum is less than 180° . Let the included side be 5 cm.

What could the measure of the third angle be? Does this measure change if the base length is changed to some other value, say 7 cm? Construct and find out.

Solution:

If the two angles of a triangle are 60° and 70° and included side is 5 cm.

Then, the third angle = $180^\circ - (60^\circ + 70^\circ) = 180^\circ - 130^\circ = 50^\circ$.



Also, If the two angles of a triangle are 60° and 70° and included side is 7 cm.
 Then, the third angle = $180^\circ - (60^\circ + 70^\circ) = 180^\circ - 130^\circ = 50^\circ$.
 Thus, changing the base length doesn't change the measure of the third angle of a triangle.

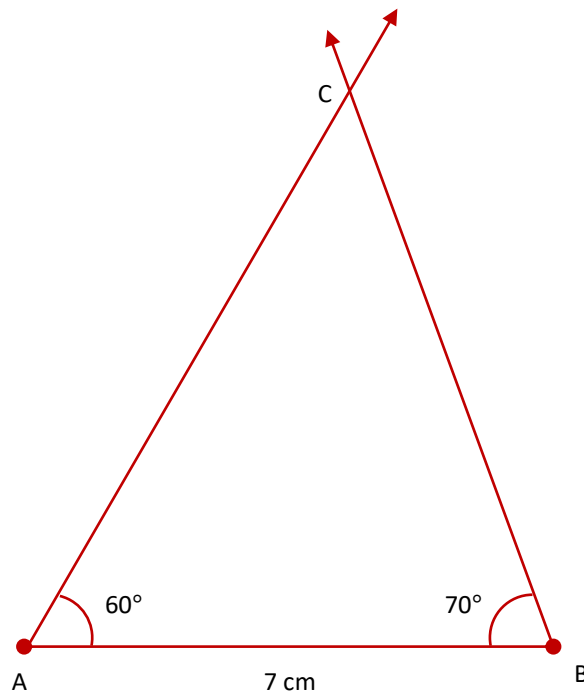


Figure it out (Page 165)

1. Find the third angle of a triangle (using a parallel line) when two of the angles are:
 - (a) 36° , 72°
 - (b) 150° , 15°
 - (c) 90° , 30°

(d) $75^\circ, 45^\circ$

Solution:

(a) $36^\circ, 72^\circ$

Since, $XY \parallel BC$, then:

$\angle ABC = \angle XAB = 36^\circ$ (Alternate interior angles)

$\angle BCA = \angle YAC = 72^\circ$ (Alternate interior angles)

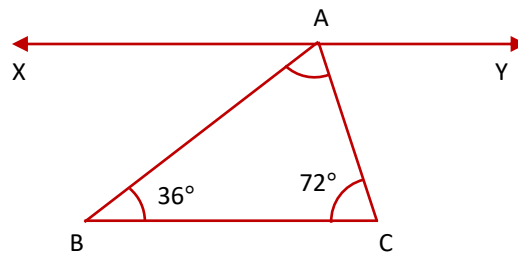
$\angle XAB + \angle BAC + \angle YAC = 180^\circ$ (Sum of angles on a straight line)

$$36^\circ + \angle BAC + 72^\circ = 180^\circ$$

$$108^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 108^\circ$$

$$\angle BAC = 72^\circ.$$



(b) $150^\circ, 15^\circ$

Since, $XY \parallel BC$, then:

$\angle ABC = \angle XAB = 150^\circ$ (Alternate interior angles)

$\angle BCA = \angle YAC = 15^\circ$ (Alternate interior angles)

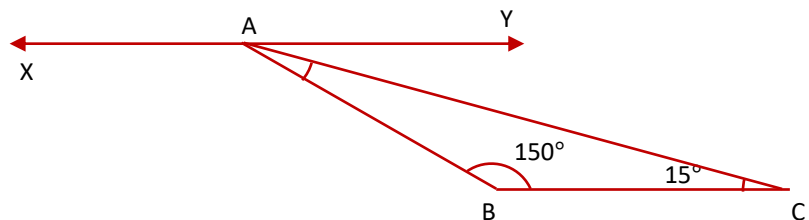
$\angle XAB + \angle BAC + \angle YAC = 180^\circ$ (Sum of angles on a straight line)

$$150^\circ + \angle BAC + 15^\circ = 180^\circ$$

$$165^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 165^\circ$$

$$\angle BAC = 15^\circ.$$



(iii) $90^\circ, 30^\circ$

Since, $XY \parallel BC$, then:

$\angle ABC = \angle XAB = 90^\circ$ (Alternate interior angles)

$\angle BCA = \angle YAC = 30^\circ$ (Alternate interior angles)

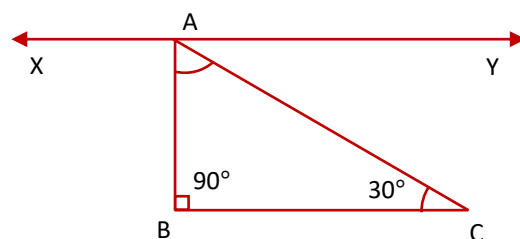
$\angle XAB + \angle BAC + \angle YAC = 180^\circ$ (Sum of angles on a straight line)

$$90^\circ + \angle BAC + 30^\circ = 180^\circ$$

$$120^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 120^\circ$$

$$\angle BAC = 60^\circ.$$



(iv) $75^\circ, 45^\circ$

Since, $XY \parallel BC$, then:

$\angle ABC = \angle XAB = 75^\circ$ (Alternate interior angles)

$\angle BCA = \angle YAC = 45^\circ$ (Alternate interior angles)

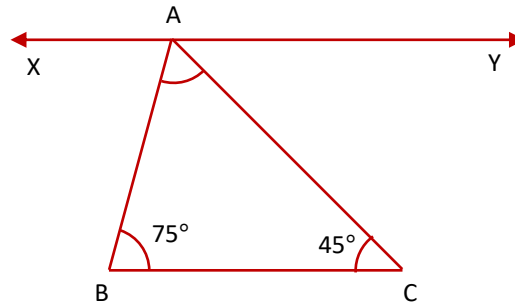
$\angle XAB + \angle BAC + \angle YAC = 180^\circ$ (Sum of angles on a straight line)

$$75^\circ + \angle BAC + 45^\circ = 180^\circ$$

$$120^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 120^\circ$$

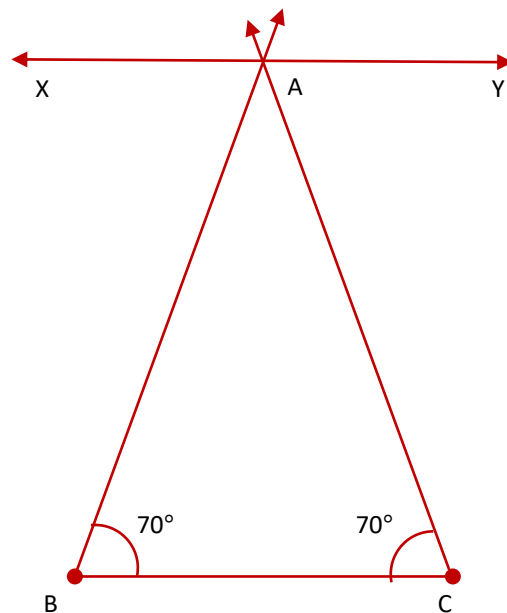
$$\angle BAC = 60^\circ.$$



2. Can you construct a triangle all of whose angles are equal to 70° ? If two of the angles are 70° what would the third angle be? If all the angles in a triangle have to be equal, then what must its measure be? Explore and find out.

Solution:

No, it is not possible to construct a triangle with all angles equal to 70° .



Let base angles, $\angle B$ and $\angle C = 70^\circ$.

Since, $XY \parallel BC$, then:

$\angle ABC = \angle XAB = 70^\circ$ (Alternate interior angles)

$\angle BCA = \angle YAC = 70^\circ$ (Alternate interior angles)

$\angle XAB + \angle BAC + \angle YAC = 180^\circ$ (Sum of angles on a straight line)

$$70^\circ + \angle BAC + 70^\circ = 180^\circ$$

$$140^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 140^\circ$$

$$\angle BAC = 40^\circ.$$

Therefore, the third angle would be 40° .

If all the angles in a triangle have to be equal, then each angle must be 60° . Such triangle with all angles equal to 60° is known as an equilateral triangle.

3. Here is a triangle in which we know $\angle B = \angle C$ and $\angle A = 50^\circ$. Can you find $\angle B$ and $\angle C$?

Solution:

Given, $\angle B = \angle C$ and $\angle A = 50^\circ$

Draw a line XY parallel to BC, then:

$\angle B = \angle XAB$ (Alternate interior angles)

$\angle C = \angle YAC$ (Alternate interior angles)

$\angle XAB + \angle A + \angle YAC = 180^\circ$ (Sum of angles on a straight line)

$\angle B + 50^\circ + \angle C = 180^\circ$

$\angle C + \angle C = 180^\circ - 50^\circ$

$2\angle C = 130^\circ$

$\angle C = 130^\circ/2 = 65^\circ$.

Therefore, $\angle B = \angle C = 65^\circ$.

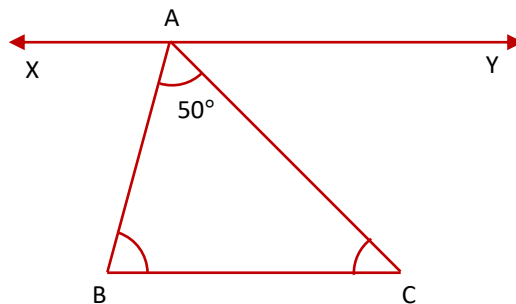


Figure it out (Page 171)

1. Construct a triangle ABC with $BC = 5\text{ cm}$, $AB = 6\text{ cm}$, $CA = 5\text{ cm}$. Construct an altitude from A to BC.

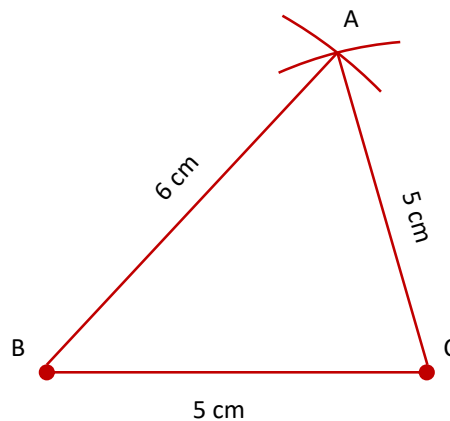
Solution:

(i) Draw the base $BC = 5\text{ cm}$.

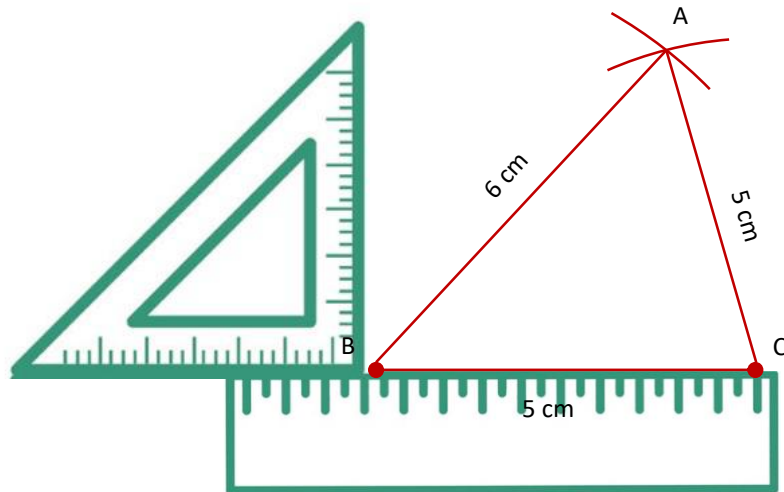
(ii) From B, draw long arc of radius 6 cm .

(iii) From C, draw arc of radius 5 cm intersecting the first arc at point A.

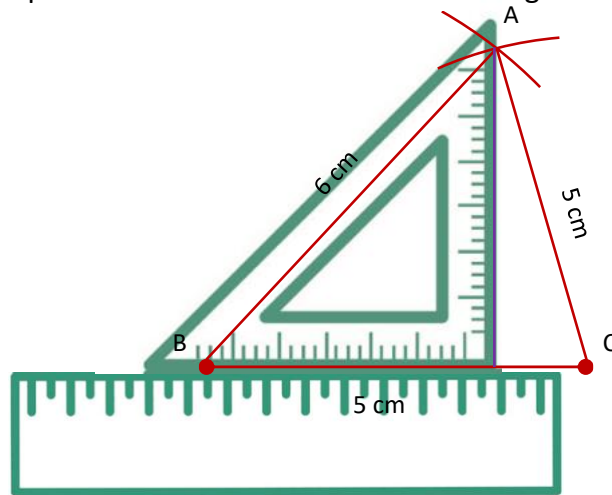
(iv) The point A is the required third vertex. Join AB and AC to get $\triangle ABC$.



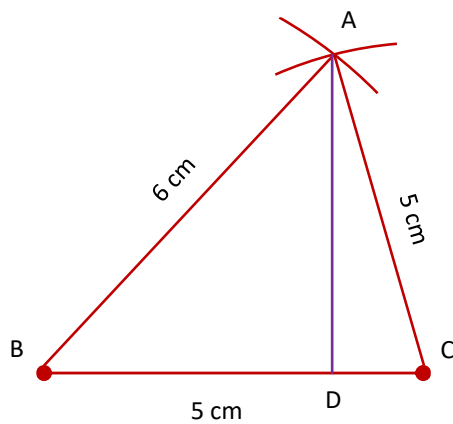
(v) Keep the ruler aligned to the base BC. Place the set square on the ruler such that one of the edges of the right angle touches the ruler.



(vi) Slide the set square along the ruler till the vertical edge of the set square touches the vertex A.



(vii) Draw the altitude to BC through A using the vertical edge of the set square.



Therefore, in $\triangle ABC$ AD is the required altitude from point A to BC.

2. Construct a triangle TRY with $RY = 4$ cm, $TR = 7$ cm, $\angle R = 140^\circ$. Construct an altitude from T to RY.

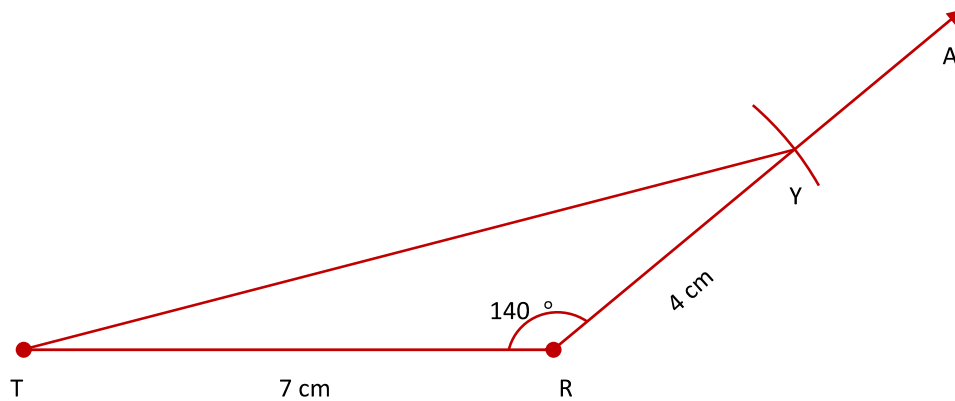
Solution:

(i) Construct side $TR = 7$ cm.

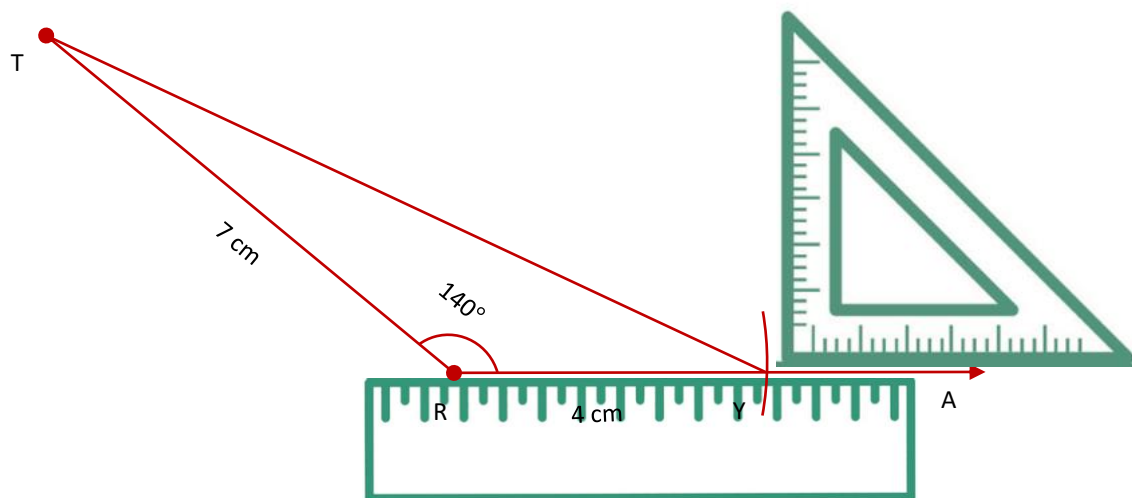
(ii) At point R, draw a ray RA making an angle of 140° with side TR.

(iii) With R as the centre and radius 4 cm, draw an arc intersecting ray RA at point Y.

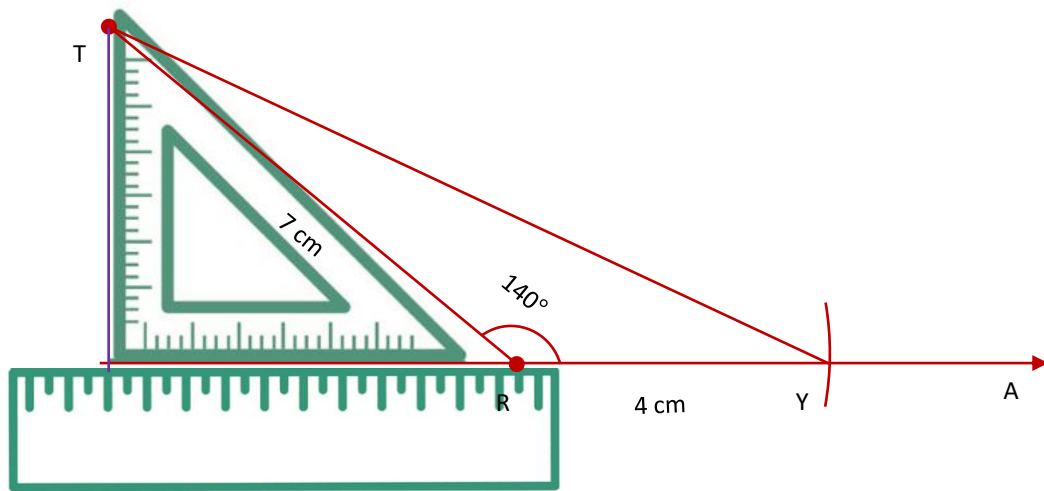
(iv) Join points T and Y to form triangle ΔTRY .



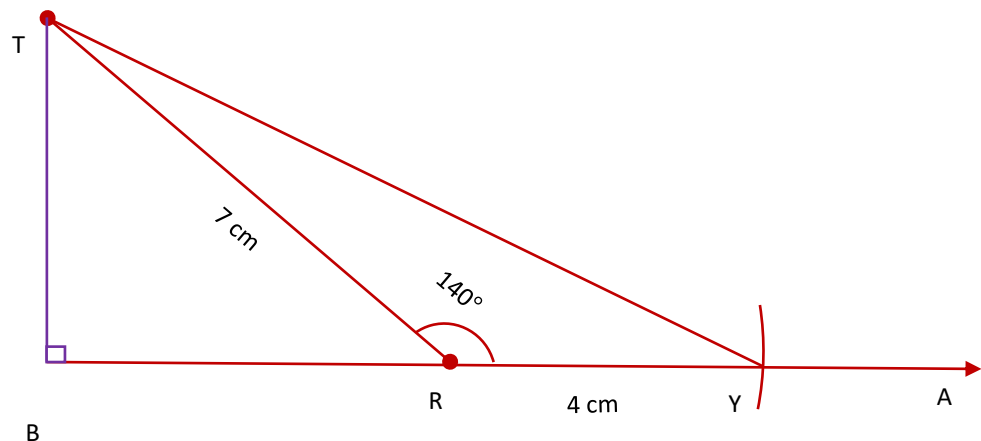
(v) Keep the ruler aligned to the side RY. Place the set square on the ruler such that one of the edges of the right angle touches the ruler.



(vi) Slide the set square along the ruler till the vertical edge of the set square touches the vertex T.



(vii) Draw the altitude to BC through A using the vertical edge of the set square.



Therefore, in $\triangle TRY$ TB is the required altitude from point T to RY.

3. Construct a right-angled triangle $\triangle ABC$ with $\angle B = 90^\circ$, $AC = 5$ cm. How many different triangles exist with these measurements?

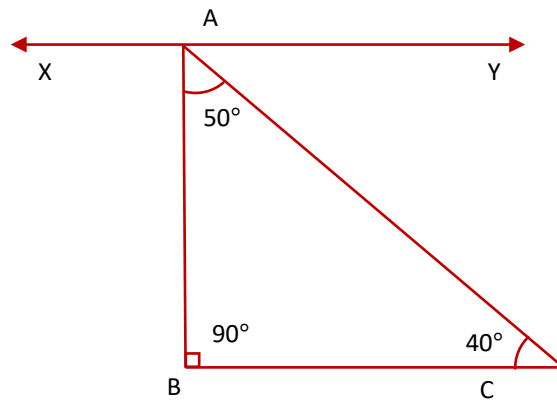
[Hint: Note that the other measurements can take any values. Take AC as the base. What values can $\angle A$ and $\angle C$ take so that the other angle is 90° ?]

Solution:

In a right-angled triangle, if $\angle B = 90^\circ$ and $AC = 5$ cm, then by the angle sum property of triangles, $\angle A + \angle C = 90^\circ$. Since $\angle A$ and $\angle C$ can take various pairs of values that sum to 90° , this results in infinitely many right-angled triangles of different shapes.

For example:

$\triangle ABC$ is a right-angled triangle with $\angle B = 90^\circ$, $AC = 5$ cm, $\angle A = 50^\circ$, and $\angle C = 40^\circ$, such that $\angle A + \angle C = 50^\circ + 40^\circ = 90^\circ$.



4. Through construction, explore if it is possible to construct an equilateral triangle that is (i) right-angled (ii) obtuse-angled. Also construct an isosceles triangle that is (i) right-angled (ii) obtuse-angled.

Solution:

An equilateral triangle has each of its angles equal to 60° , so it is impossible to construct a right-angled or obtuse-angled equilateral triangle.

An isosceles triangle can be right-angled, with one angle of 90° and the other two angles of 45° each.

An isosceles triangle can also be obtuse-angled, with one angle of 100° and the other two angles of 40° each.

