

Chapter 8: Working with Fractions

Figure it Out (Page 176 – 177)

1. Tenzin drinks $\frac{1}{2}$ glass of milk every day. How many glasses of milk does he drink in a week?
How many glasses of milk did he drink in the month of January?

Solution:

Glass of milk drank every day = $\frac{1}{2}$

Glasses of milk drank in a week = $\frac{1}{2} \times 7 = \frac{7}{2}$

Days in month of January = 31

Glasses of milk drank in month of January = $31 \times \frac{1}{2} = \frac{31}{2} = 15\frac{1}{2}$.

2. A team of workers can make 1 km of a water canal in 8 days. So, in one day, the team can make ____ km of the water canal. If they work 5 days a week, they can make ____ km of the water canal in a week.

Solution:

The length of canal made by workers in 8 days = 1 km

The length of canal made by workers in 1 day = $\frac{1}{8}$ km

By working 5 days in a week, the length of canal made by workers = $5 \times \frac{1}{8} = \frac{5}{8}$ km.

3. Manju and two of her neighbours buy 5 litres of oil every week and share it equally among the 3 families. How much oil does each family get in a week? How much oil will one family get in 4 weeks?

Solution:

Amount of oil shared by 3 families in a week = 5 litres

Amount of oil one family got in a week = $\frac{5}{3}$ litres

Amount of oil one family got in 4 weeks = $4 \times \frac{5}{3} = \frac{20}{3} = 6\frac{2}{3}$ litres.

4. Safia saw the Moon setting on Monday at 10 pm. Her mother, who is a scientist, told her that every day the Moon sets $\frac{5}{6}$ hour later than the previous day. How many hours after 10 pm will the moon set on Thursday?

Solution:

No. of days from Monday to Thursday = 3

Delay in moon setting time per day = $\frac{5}{6}$ hour

Delay in moon setting time over 3 days = $3 \times \frac{5}{6}$ hour = $\frac{5}{2} = 2.5$ hours

Thus, the Moon will set 2.5 hours or 2 hours 30 minutes after 10 PM on Thursday.

5. Multiply and then convert it into a mixed fraction:

(a) $7 \times \frac{3}{5}$

(b) $4 \times \frac{1}{3}$

(c) $\frac{9}{7} \times 6$

(d) $\frac{13}{11} \times 6$

Solution:

(a) $7 \times \frac{3}{5} = \frac{21}{5} = 4 \frac{1}{5}$.

(b) $4 \times \frac{1}{3} = \frac{4}{3}$.

(c) $\frac{9}{7} \times 6 = \frac{54}{7}$.

(d) $\frac{13}{11} \times 6 = \frac{78}{11}$.

Figure it Out (Page 180 – 181)

1. Find the following products. Use a unit square as a whole for representing the fractions:

(a) $\frac{1}{3} \times \frac{1}{5}$

(b) $\frac{1}{4} \times \frac{1}{3}$

(c) $\frac{1}{5} \times \frac{1}{2}$

(d) $\frac{1}{6} \times \frac{1}{5}$

Solution:

(a) $\frac{1}{3} \times \frac{1}{5}$

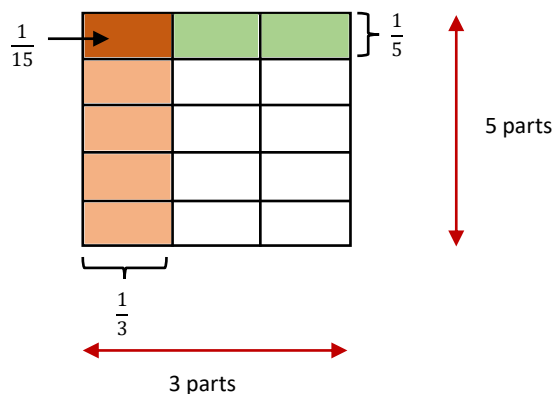
$\frac{1}{3}$ (multiplier) $\times \frac{1}{5}$ (multiplicand)

Number of rows = Denominator of the multiplicand = 5

Number of columns = Denominator of the multiplier = 3

Thus, the whole is divided into $(5 \times 3) = 15$ equal parts

So, $\frac{1}{3} \times \frac{1}{5} = \frac{1}{3 \times 5} = \frac{1}{15}$.



(b) $\frac{1}{4} \times \frac{1}{3}$

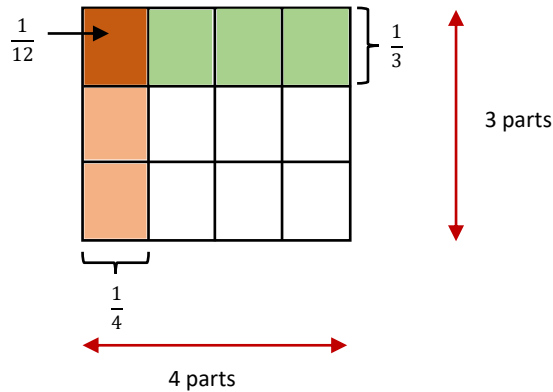
$\frac{1}{4}$ (multiplier) \times $\frac{1}{3}$ (multiplicand)

Number of rows = Denominator of the multiplicand = 3

Number of columns = Denominator of the multiplier = 4

Thus, the whole is divided into $(3 \times 4) = 12$ equal parts

So, $\frac{1}{4} \times \frac{1}{3} = \frac{1}{4 \times 3} = \frac{1}{12}$.



(c) $\frac{1}{5} \times \frac{1}{2}$

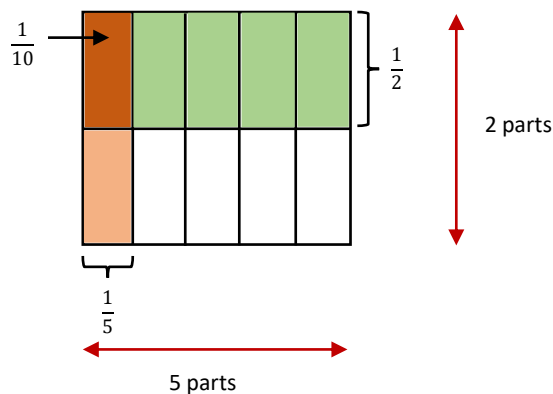
$\frac{1}{5}$ (multiplier) \times $\frac{1}{2}$ (multiplicand)

Number of rows = Denominator of the multiplicand = 2

Number of columns = Denominator of the multiplier = 5

Thus, the whole is divided into $(2 \times 5) = 10$ equal parts

So, $\frac{1}{5} \times \frac{1}{2} = \frac{1}{5 \times 2} = \frac{1}{10}$.



(d) $\frac{1}{6} \times \frac{1}{5}$

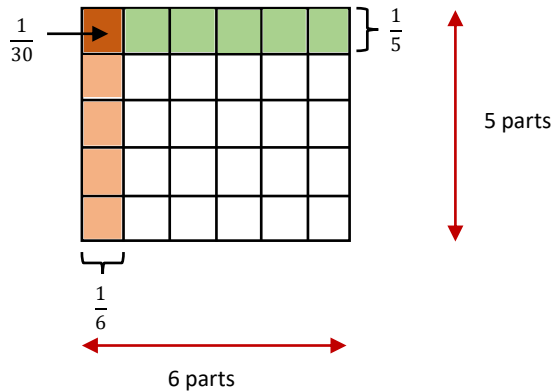
$\frac{1}{6}$ (multiplier) \times $\frac{1}{5}$ (multiplicand)

Number of rows = Denominator of the multiplicand = 5

Number of columns = Denominator of the multiplier = 6

Thus, the whole is divided into $(5 \times 6) = 30$ equal parts

So, $\frac{1}{6} \times \frac{1}{5} = \frac{1}{6 \times 5} = \frac{1}{30}$.



2. Find the following products. Use a unit square as a whole for representing the fractions and carrying out the operations.

(a) $\frac{2}{3} \times \frac{4}{5}$

(b) $\frac{1}{4} \times \frac{2}{3}$

(c) $\frac{3}{5} \times \frac{1}{2}$

(d) $\frac{4}{6} \times \frac{3}{5}$

Solution:

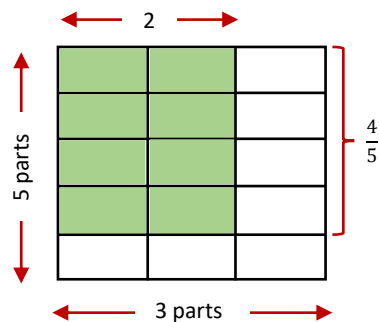
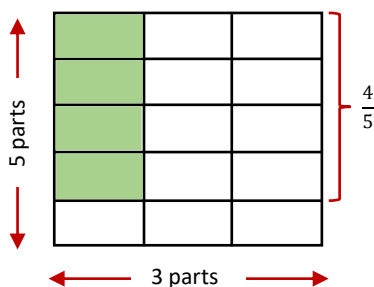
(a) $\frac{2}{3} \times \frac{4}{5}$

First, the whole is divided into 5 rows and 3 columns creating 15 (5×3) equal parts.

The value we get by dividing $\frac{4}{5}$ into 3 equal parts is $\frac{4}{3 \times 5}$.

Thus, we multiply this result by 2 to get the product. This is $\frac{2 \times 4}{3 \times 5}$.

So, $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$.

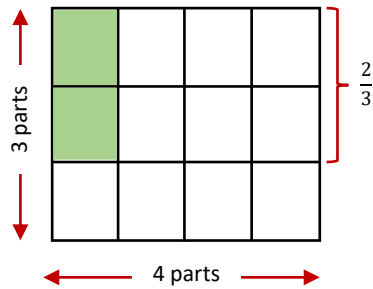


(b) $\frac{1}{4} \times \frac{2}{3}$

First, the whole is divided into 3 rows and 4 columns creating 12 (3×4) equal parts.

The value we get by dividing $\frac{2}{3}$ into 4 equal parts is $\frac{2}{4 \times 3}$.

So, $\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3} = \frac{2}{12}$.



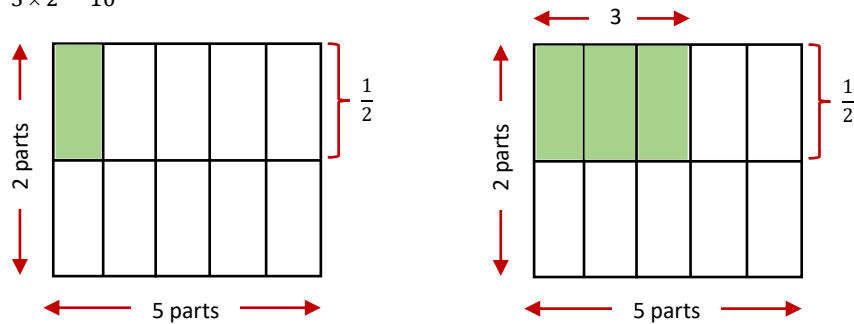
(c) $\frac{3}{5} \times \frac{1}{2}$

First, the whole is divided into 2 rows and 5 columns creating 10 (2×5) equal parts.

The value we get by dividing $\frac{1}{2}$ into 5 equal parts is $\frac{1}{5 \times 2}$.

Thus, we multiply this result by 3 to get the product. This is $\frac{3 \times 1}{5 \times 2}$.

So, $\frac{3}{5} \times \frac{1}{2} = \frac{3 \times 1}{5 \times 2} = \frac{3}{10}$.



(d) $\frac{4}{6} \times \frac{3}{5}$

First, the whole is divided into 5 rows and 6 columns creating 30 (5×6) equal parts.

The value we get by dividing $\frac{3}{5}$ into 6 equal parts is $\frac{3}{6 \times 5}$.

Thus, we multiply this result by 4 to get the product. This is $\frac{4 \times 3}{6 \times 5}$.

So, $\frac{4}{6} \times \frac{3}{5} = \frac{4 \times 3}{6 \times 5} = \frac{12}{30}$.

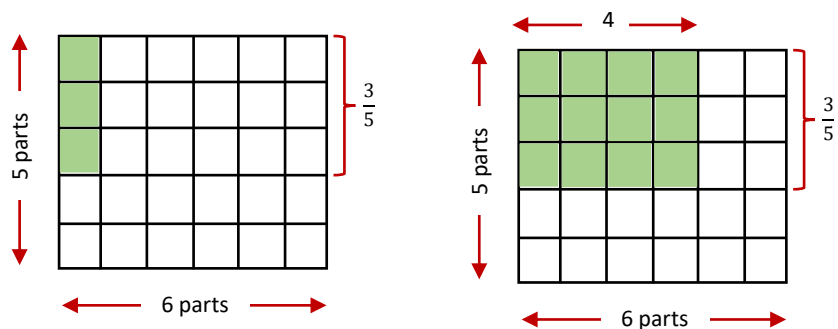


Figure it out (Page 183)

1. A water tank is filled from a tap. If the tap is open for 1 hour, $\frac{7}{10}$ of the tank gets filled.

How much of the tank is filled if the tap is open for

(a) $\frac{1}{3}$ hour _____

(b) $\frac{2}{3}$ hour _____

(c) $\frac{3}{4}$ hour _____

(d) $\frac{7}{10}$ hour _____

(e) For the tank to be full, how long should the tap be running?

Solution:

Part of the tank filled in 1 hour = $\frac{7}{10}$

(a) Part of the tank filled in $\frac{1}{3}$ hour = $\frac{1}{3} \times \frac{7}{10} = \frac{7}{30}$.

(b) Part of the tank filled in $\frac{2}{3}$ hour = $\frac{2}{3} \times \frac{7}{10} = \frac{14}{30} = \frac{7}{15}$.

(c) Part of the tank filled in $\frac{3}{4}$ hour = $\frac{3}{4} \times \frac{7}{10} = \frac{21}{40}$.

(d) Part of the tank filled in $\frac{7}{10}$ hour = $\frac{7}{10} \times \frac{7}{10} = \frac{49}{100}$.

(e) Part of the tank filled in 1 hour = $\frac{7}{10}$.

or Time required to fill $\frac{7}{10}$ of a tank = 1 hour.

Time required to fill 1 tank = $1 \div \frac{7}{10} = 1 \times \frac{10}{7} = \frac{10}{7}$ hours = $1\frac{3}{7}$ hours.

2. The government has taken $\frac{1}{6}$ of Somu's land to build a road. What part of the land remains with Somu now? She gives half of the remaining part of the land to her daughter, Krishna, and $\frac{1}{3}$ it to her son Bora. After giving them their shares, she kept the remaining land for herself.

(a) What part of the original land did Krishna get?

(b) What part of the original land did Bora get?

(c) What part of the original land did Somu keep for herself?

Solution:

Part of Somu's land acquired by the government = $\frac{1}{6}$

Somu's original land = $1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$

(a) Part of the land given to Krishna = $\frac{1}{2}$ of original land = $\frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$.

(b) Part of the land given to Bora = $\frac{1}{3}$ of original land = $\frac{1}{3} \times \frac{5}{6} = \frac{5}{18}$.

$$\begin{aligned}
 \text{(c) Part of the land Somu kept for herself} &= \frac{5}{6} - \left(\frac{5}{12} + \frac{5}{18} \right) \\
 &= \frac{5}{6} - \left(\frac{3 \times 5 + 5 \times 2}{36} \right) \\
 &= \frac{5}{6} - \left(\frac{15 + 10}{36} \right) \\
 &= \frac{5}{6} - \left(\frac{25}{36} \right) \\
 &= \frac{6 \times 5 - 25}{36} \\
 &= \frac{30 - 25}{36} = \frac{5}{36}.
 \end{aligned}$$

3. Find the area of a rectangle of sides $3\frac{3}{4}$ ft and $9\frac{3}{5}$ ft.

Solution:

$$\text{Area of rectangle} = 3\frac{3}{4} \text{ ft} \times 9\frac{3}{5} \text{ ft} = \frac{15}{4} \text{ ft} \times \frac{48}{5} \text{ ft}$$

$$\begin{array}{r}
 3 \qquad 12 \\
 = \frac{15}{4} \times \frac{48}{5} = 3 \times 12 = 36 \text{ sq ft} \\
 1 \qquad 1
 \end{array}$$

4. Tsewang plants four saplings in a row in his garden. The distance between two saplings is $\frac{3}{4}$ m. Find the distance between the first and last sapling.

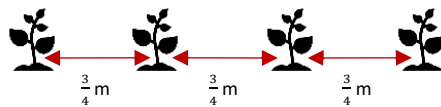
[Hint: Draw a rough diagram with four saplings with distance between two saplings as $\frac{3}{4}$ m]

Solution:

No. of saplings in a row in garden = 4

Distance between two saplings = $\frac{3}{4}$

$$\text{Distance between first and last sapling} = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{3+3+3}{4} = \frac{9}{4} \text{ m} = 2\frac{1}{4} \text{ m}.$$



5. Which is heavier: $\frac{12}{15}$ of 500 grams or $\frac{3}{20}$ of 4 kg?

Solution:

$$\frac{12}{15} \text{ of 500 grams} = \frac{12}{15} \times 500 \text{ g}$$

$$\begin{array}{r}
 4 \qquad 100 \\
 = \frac{12}{15} \times 500 = 4 \times 100 = 400 \text{ g.} \\
 3 \qquad 1
 \end{array}$$

$$\frac{3}{20} \text{ of } 4 \text{ kg} = \frac{3}{20} \times 4000 \text{ g}$$

$$= \frac{3}{\cancel{20}^1} \times \frac{200}{\cancel{4000}^{200}} = 3 \times 200 = 600 \text{ g.}$$

Hence, $\frac{3}{20}$ of 4 kg is heavier than $\frac{12}{15}$ of 500 grams.

Textbook Page 185

Is the Product Always Greater than the Numbers Multiplied?

Q. What can you conclude about the relationship between the numbers multiplied and the product? Fill in the blanks:

- When one of the numbers being multiplied is between 0 and 1, the product is _____ (greater/less) than the other number.
- When one of the numbers being multiplied is greater than 1, the product is _____ (greater/less) than the other number.

Solution:

(i) When one of the numbers being multiplied is between 0 and 1, the product is less than the other number.

Example:

Let one number be $\frac{1}{4}$ and other number be 100.

$$\text{Product} = \frac{1}{4} \times 100 = 25.$$

Hence, the product (25) is less than the other number (100).

(ii) When one of the numbers being multiplied is greater than 1, the product is greater than the other number.

Example:

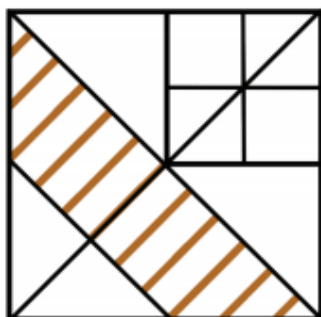
Let one number be 10 and other number 50.

$$\text{Product} = 10 \times 50 = 500.$$

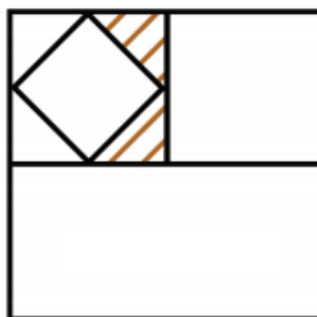
Hence, the product (500) is greater than the other number (50).

Fractional Relations

Q. In each of the figures given below, find the fraction of the big square that the shaded region occupies.



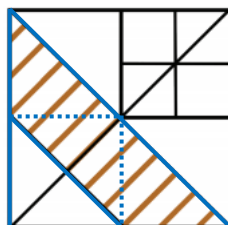
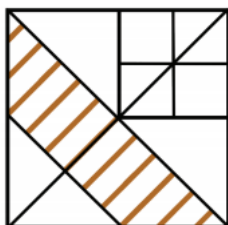
(i)



(ii)

Solution:

(i)



There are 4 triangles in half of the area of the whole square.

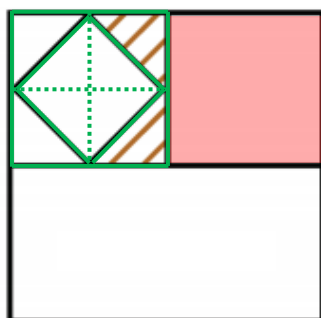
So, there are total 8 triangles in the whole square.

Area of each triangle = $\frac{1}{8}$.

Area of the shaded region = 3 triangles = $3 \times \frac{1}{8} = \frac{3}{8}$.

Hence, the shaded region occupies $\frac{3}{8}$ area of the whole square.

(ii)



There are total number of 4 red small squares in the area of the whole square.

Total number of triangles in one small square = 8

So, total number of triangles in the area of the whole square = $4 \times 8 = 32$.

Area of each triangle = $\frac{1}{32}$

Area of the shaded region = 2 triangles = $2 \times \frac{1}{32} = \frac{2}{32} = \frac{1}{16}$.

Hence, the shaded region occupies $\frac{1}{16}$ area of the whole square.

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Q. If we assume 1 gold dinar = 12 silver drammās, 1 silver dramma = 4 copper panas, 1 copper pana = 6 mashakas, and 1 pana = 30 cowrie shells,

$$1 \text{ copper pana} = \frac{1}{48} \text{ gold dinar } \left(\frac{1}{12} \times \frac{1}{4} \right)$$

$$1 \text{ cowrie shell} = \frac{1}{30} \text{ copper panas}$$

$$1 \text{ cowrie shell} = \frac{1}{1440} \text{ gold dinar.}$$

Solution:

$$1 \text{ copper pana} = \frac{1}{48} \text{ gold dinar}$$

$$1 \text{ copper pana} = 30 \text{ cowrie shells}$$

$$\text{or } 30 \text{ cowrie shells} = 1 \text{ copper pana}$$

$$\text{or } 1 \text{ cowrie shell} = \frac{1}{30} \text{ copper panas.}$$

$$\text{or } 1 \text{ cowrie shell} = \frac{1}{30} \times \frac{1}{48} \text{ gold dinar} = \frac{1}{1440} \text{ gold dinar.}$$

Figure it out (Page 196)

1. Evaluate the following:

$3 \div \frac{7}{9}$	$\frac{14}{4} \div 2$	$\frac{2}{3} \div \frac{2}{3}$	$\frac{14}{6} \div \frac{7}{3}$
$\frac{4}{3} \div \frac{3}{4}$	$\frac{7}{4} \div \frac{1}{7}$	$\frac{8}{2} \div \frac{4}{15}$	
$\frac{1}{5} \div \frac{1}{9}$	$\frac{1}{6} \div \frac{11}{12}$	$3\frac{2}{3} \div 1\frac{3}{8}$	

Solution:

$$(i) 3 \div \frac{7}{9} = 3 \times \frac{9}{7} = \frac{27}{7} = 3\frac{6}{7}.$$

$$(ii) \frac{4}{3} \div \frac{3}{4} = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9} = 1\frac{7}{9}.$$

$$(iii) \frac{1}{5} \div \frac{1}{9} = \frac{1}{5} \times \frac{9}{1} = \frac{9}{5} = 1\frac{4}{5}.$$

$$(iv) \frac{14}{4} \div 2 = \frac{14}{4} \times \frac{1}{2} = \frac{7}{4} = 1\frac{3}{4}.$$

$$(v) \frac{7}{4} \div \frac{1}{7} = \frac{7}{4} \times \frac{7}{1} = \frac{49}{4} = 12\frac{1}{4}.$$

$$(vi) \frac{1}{6} \div \frac{11}{12} = \frac{1}{6} \times \frac{12}{11} = \frac{2}{11}.$$

$$(vii) \frac{2}{3} \div \frac{2}{3} = \frac{2}{3} \times \frac{3}{2} = 1.$$

$$(viii) \frac{8}{2} \div \frac{4}{15} = \frac{8}{2} \times \frac{15}{4} = 15.$$

$$(ix) 3\frac{2}{3} \div 1\frac{3}{8} = \frac{11}{3} \div \frac{11}{8} = \frac{11}{3} \times \frac{8}{11} = \frac{8}{3} = 2\frac{2}{3}.$$

$$(x) \frac{14}{6} \div \frac{7}{3} = \frac{14}{6} \times \frac{3}{7} = 1.$$

2. For each of the questions below, choose the expression that describes the solution. Then simplify it.

(a) Maria bought 8 m of lace to decorate the bags she made for school. She used $\frac{1}{4}$ m for each bag and finished the lace. How many bags did she decorate?

(i) $8 \times \frac{1}{4}$

(ii) $\frac{1}{8} \times \frac{1}{4}$

(iii) $8 \div \frac{1}{4}$

(iv) $\frac{1}{4} \div 8$

Solution:

Lace bought = 8 m

Lace used to decorate one bag = $\frac{1}{4}$ m

No. of bags decorated = $8 \div \frac{1}{4}$.

So, (iii) $8 \div \frac{1}{4}$ is the correct answer.

(b) $\frac{1}{2}$ metre of ribbon is used to make 8 badges. What is the length of the ribbon used for each badge?

(i) $8 \times \frac{1}{2}$

(ii) $\frac{1}{2} \div \frac{1}{8}$

(iii) $8 \div \frac{1}{2}$

(iv) $\frac{1}{2} \div 8$

Solution:

Length of ribbon used to make 8 badges = $\frac{1}{2}$ metres.

Length of ribbon used to make each badge = $\frac{1}{2} \div 8$.

So, (iv) $\frac{1}{2} \div 8$ is the correct answer.

(c) A baker needs $\frac{1}{6}$ kg of flour to make one loaf of bread. He has 5 kg of flour. How many loaves of bread can he make?

(i) $5 \times \frac{1}{6}$

(ii) $\frac{1}{6} \div 5$

(iii) $5 \div \frac{1}{6}$

(iv) 5×6

Solution:

Quantity of flour needed to make 1 loaf of bread = $\frac{1}{6}$ kg

Total quantity of flour = 5 kg

No. of loafs of bread made = $5 \div \frac{1}{6}$.

So, (iii) $5 \div \frac{1}{6}$ is the correct answer.

4. Pāṭiṅgaṇita, a book written by Sridharacharya in the 9th century CE, mentions this problem:

“Friend, after thinking, what sum will be obtained by adding together $1 \div \frac{1}{6}$, $1 \div \frac{1}{10}$, $1 \div \frac{1}{13}$, $1 \div \frac{1}{9}$, and $1 \div \frac{1}{2}$ ”. What should the friend say?

Solution:

The sum obtained = $\left(1 \div \frac{1}{6}\right) + \left(1 \div \frac{1}{10}\right) + \left(1 \div \frac{1}{13}\right) + \left(1 \div \frac{1}{9}\right) + \left(1 \div \frac{1}{2}\right)$

$$= \left(1 \times \frac{6}{1}\right) + \left(1 \times \frac{10}{1}\right) + \left(1 \times \frac{13}{1}\right) + \left(1 \times \frac{9}{1}\right) + \left(1 \times \frac{2}{1}\right)$$

$$= 6 + 10 + 13 + 9 + 2 = 40.$$

Therefore, the friend should say that the sum is 40.

5. Mira is reading a novel that has 400 pages. She read $\frac{1}{5}$ of the pages yesterday and $\frac{3}{10}$ of the pages today. How many more pages does she need to read to finish the novel?

Solution:

The total number of pages in the novel = 400.

The number of pages read by Mira yesterday = $\frac{1}{5}$ of 400 = $\frac{1}{5} \times 400 = \frac{400}{5} = 80$ pages.

The number of pages read by Mira today = $\frac{3}{10}$ of 400 = $\frac{3}{10} \times 400 = \frac{1200}{10} = 120$ pages.

Total pages read by Mira = $80 + 120 = 200$ pages.

Therefore, the number of pages she needs to finish the novel = $400 - 200 = 200$ pages.

6. A car runs 16 km using 1 litre of petrol. How far will it go using $2\frac{3}{4}$ litres of petrol?

Solution:

Distance travelled by car in 1 litre of petrol = 16 km.

Distance travelled by car in $2\frac{3}{4}$ litres of petrol = $16 \times 2\frac{3}{4} = 16 \times \frac{11}{4} = 4 \times 11 = 44$ km.

7. Amritpal decides on a destination for his vacation. If he takes a train, it will take him $5\frac{1}{6}$ hours to get there. If he takes a plane, it will take him $\frac{1}{2}$ hour. How many hours does the plane save?

Solution:

The time taken by the train to reach the destination = $5\frac{1}{6}$ hours.

The time taken by the plane to reach the destination = $\frac{1}{2}$ hour.

The time saved by travelling by plane = $5\frac{1}{6} - \frac{1}{2}$

$$= \frac{31}{6} - \frac{1}{2} = \frac{31-3}{6} = \frac{28}{6} = \frac{14}{3} = 4\frac{2}{3} \text{ hours.}$$

8. Mariam's grandmother baked a cake. Mariam and her cousins finished $\frac{4}{5}$ of the cake. The remaining cake was shared equally by Mariam's three friends. How much of the cake did each friend get?

Solution:

Part of cake finished by Mariam and her cousins = $\frac{4}{5}$

Part of cake left = $1 - \frac{4}{5} = \frac{5-4}{5} = \frac{1}{5}$

This $\frac{1}{5}$ part of the cake is shared equally among 3 friends.

Part each friend got = $\frac{1}{5} \div 3 = \frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$.

Thus, each of Mariam's three friends got $\frac{1}{15}$ of the cake.

9. Choose the option(s) describing the product of $(\frac{565}{465} \times \frac{707}{676})$:

(a) $> \frac{565}{465}$

$$(b) < \frac{565}{465}$$

$$(c) > \frac{676}{707}$$

$$(d) < \frac{707}{676}$$

$$(e) > 1$$

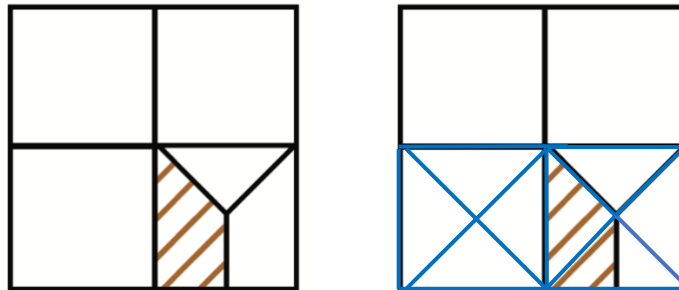
$$(f) < 1$$

Solution:

$$\frac{565}{465} > 1 \text{ and } \frac{707}{676} > 1$$

Since, both numbers are greater than 1, their product is greater than both the numbers.
Therefore, (a), (c) and (e) are correct options.

10. What fraction of the whole square is shaded?



Solution:

The area of the bigger square is divided into four equal smaller squares.

The number of triangles in a smaller square = 4

Therefore, the total number of triangles in the bigger square = $4 \times 4 = 16$

Area of each triangle = $\frac{1}{16}$

Area of the shaded region = area of $1\frac{1}{2}$ triangle = $1\frac{1}{2} \times \frac{1}{16} = \frac{3}{2} \times \frac{1}{16} = \frac{3}{32}$.

Therefore, $\frac{3}{32}$ fraction of the whole square is shaded.

11. A colony of ants set out in search of food. As they search, they keep splitting equally at each point (as shown in the Fig. 8.7) and reach two food sources, one near a mango tree and another near a sugarcane field. What fraction of the original group reached each food source?

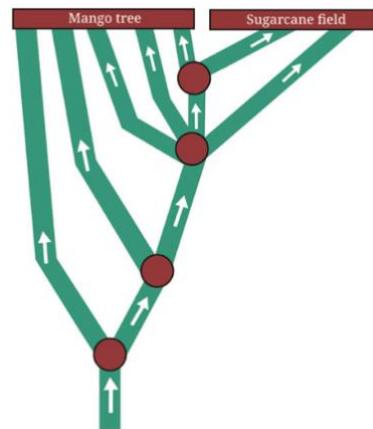
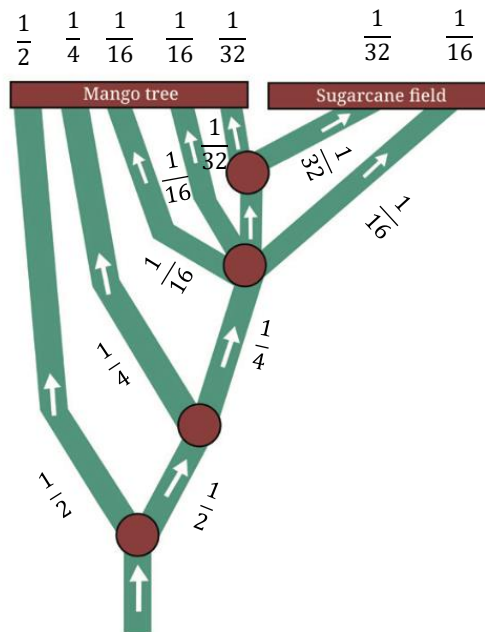


Fig. 8.7

Solution:



At first point ants split into two ways. So, fraction of ants at each way = $1 \div 2 = \frac{1}{2}$.

At second point ants split further into two ways. So, fraction of ants at each way = $\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

At third point ants split further into four ways. So, fraction of ants at each way = $\frac{1}{4} \div 4 = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$.

At fourth point ants split further into two ways. So, fraction of ants at each way = $\frac{1}{16} \div 2 = \frac{1}{32}$
 $\times \frac{1}{2} = \frac{1}{32}$.

Fraction of ants reaching mango tree = $\frac{1}{2} + \frac{1}{4} + \frac{1}{16} + \frac{1}{16} + \frac{1}{32} = \frac{16+8+2+2+1}{32} = \frac{29}{32}$.

Fraction of ants reaching sugarcane field = $\frac{1}{16} + \frac{1}{32} = \frac{2+1}{32} = \frac{3}{32}$.

12. What is $\left(1 - \frac{1}{2}\right)$?

$\left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right)$?

$\left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{5}\right)$?

$\left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{5}\right) \times \left(1 - \frac{1}{6}\right) \times \left(1 - \frac{1}{7}\right) \times \left(1 - \frac{1}{8}\right) \times \left(1 - \frac{1}{9}\right) \times \left(1 - \frac{1}{10}\right)$?

Make a general statement and explain.

Solution:

$$(i) \left(1 - \frac{1}{2}\right) = \frac{2-1}{2} = \frac{1}{2}.$$

$$(ii) \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) = \left(\frac{2-1}{2}\right) \times \left(\frac{3-1}{3}\right) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}.$$

$$(iii) \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{5}\right) = \left(\frac{2-1}{2}\right) \times \left(\frac{3-1}{3}\right) \times \left(\frac{4-1}{4}\right) \times \left(\frac{5-1}{5}\right)$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{1}{5}.$$

$$(iv) \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{5}\right) \times \left(1 - \frac{1}{6}\right) \times \left(1 - \frac{1}{7}\right) \times \left(1 - \frac{1}{8}\right) \times \left(1 - \frac{1}{9}\right) \times \left(1 - \frac{1}{10}\right)$$

$$= \left(\frac{2-1}{2}\right) \times \left(\frac{3-1}{3}\right) \times \left(\frac{4-1}{4}\right) \times \left(\frac{5-1}{5}\right) \times \left(\frac{6-1}{6}\right) \times \left(\frac{7-1}{7}\right) \times \left(\frac{8-1}{8}\right) \times \left(\frac{9-1}{9}\right) \times \left(\frac{10-1}{10}\right)$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8} \times \frac{8}{9} \times \frac{9}{10} = \frac{1}{10}.$$

General statement:

$$\left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{5}\right) \dots \dots \dots \left(1 - \frac{1}{n}\right) = \frac{1}{n}.$$

